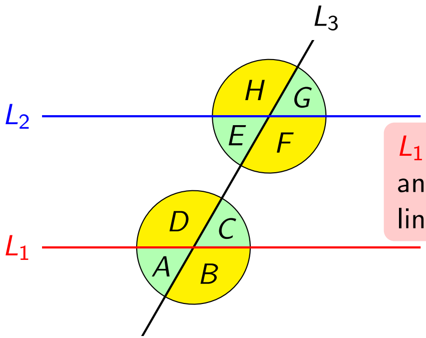
Trigonometry DLA Series

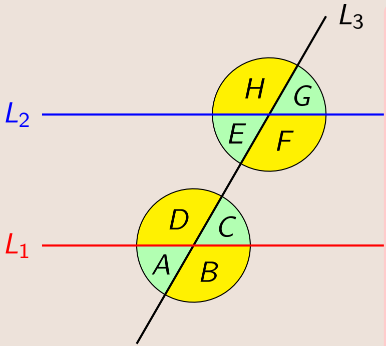


L_1 and L_2 are parallel lines and L_3 is the transverse line.

Vertical & Alternate Angles

In this DLA, we look at the angles when two parallel lines are crossed by a transverse line.

Assuming L_1 and L_2 are **parallel lines** and L_3 is the **transverse line**, opposite angles with common vertex are called vertical angles. Vertical angles are congruent.



Angles A and C are called vertical angles

and $m\angle A = m\angle C$

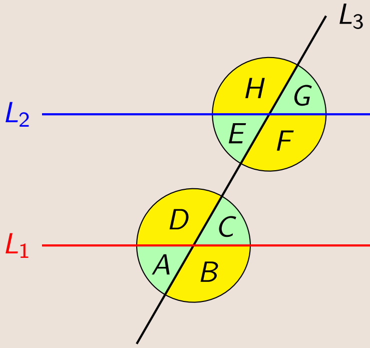
Angles B and D are called vertical angles

and $m\angle B = m\angle D$

Similarly $m\angle E = m\angle G$

and $m\angle F = m\angle H$

Assuming L_1 and L_2 are **parallel lines** and L_3 is the **transverse line**, corresponding angles are formed at the same relative position at each intersection. Corresponding angles are congruent.



Angles A and E are corresponding angles

and $m\angle A = m\angle E$

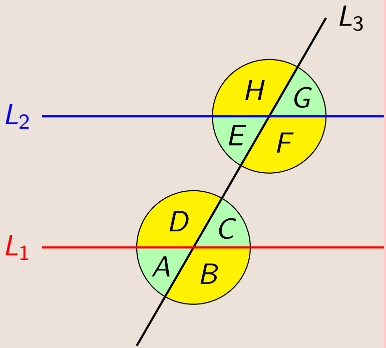
Angles B and F are called corresponding angles

and $m\angle B = m\angle F$

Similarly $m\angle D = m\angle H$

and $m\angle C = m\angle G$

An angle whose sides lie in opposite directions from the vertex in the same straight line is called a straight or flat angle.



Angles A and B form a straight angle

and $m\angle A + m\angle B = 180^\circ$.

Angles B and C form a straight angle

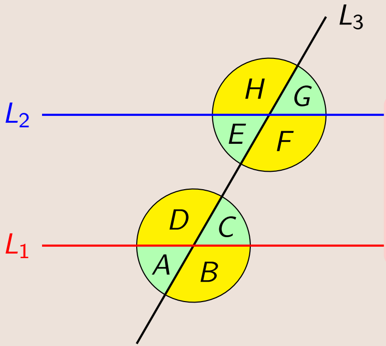
and $m\angle B + m\angle C = 180^\circ$.

Angles E and F form a straight angle

and $m\angle E + m\angle F = 180^\circ$.

and so on.

Alternate angles are shaped by the two parallel lines crossed by a transverse line. Assuming L_1 and L_2 are **parallel lines**, when the **transverse line** L_3 crosses them, some angles are formed. Those angles are known as interior or exterior angles.

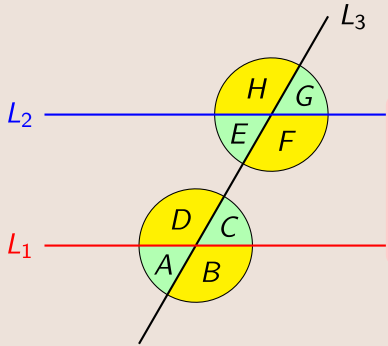


Angles C, D, E, and F are called interior angles.

Angles A, B, G, and H are called exterior angles.

Alternate interior angles are the pair of angles that are formed on the inner side of the two parallel lines but on the opposite side of the transverse line.

Alternate interior angles are congruent.

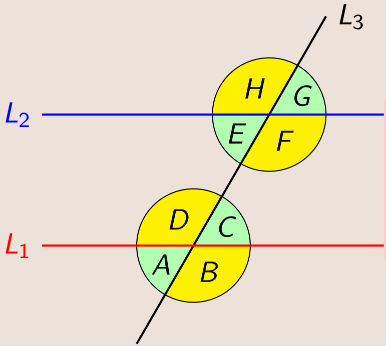


Angles C, D, E, and F are called interior angles.

Angles A, B, G, and H are called exterior angles.

Alternate exterior angles are the pair of angles that are formed on the outer side of the parallel lines but on the opposite side of the transverse line.

Alternate exterior angles are congruent.

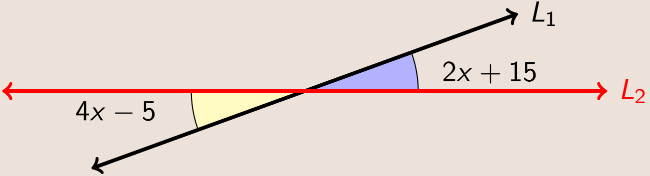


Angles C, D, E, and F are called interior angles.

Angles A, B, G, and H are called exterior angles.

Example:

Find x using the marked angles, then find the measure of each angle.



Solution:

The marked angles are vertical angles, therefore they are congruent.

Solution(continued):

$$
\begin{aligned}
4x - 5 &= 2x + 15 && \text{(Vertical angles are equal.)} \\
4x - 2x &= 15 + 5 && \text{(Equation Properties)} \\
2x &= 20 && \text{(Simplify)} \\
x &= 10 && \text{(Division Property)}
\end{aligned}
$$

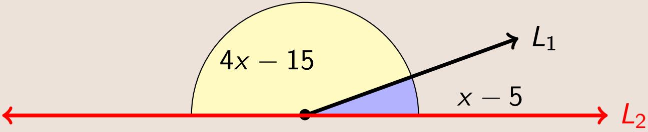
Now to measure each angle, we simply evaluate each one with $x = 10$.

$$
\begin{aligned}
4x - 5 &= 2x + 15 \\
4(10) - 5 &= 2(10) + 15 \\
40 - 5 &= 20 + 15 \\
35 &= 35
\end{aligned}
$$

Each marked angle is 35°.

Example:

Find x using the marked angles, then find the measure of each angle.



Solution:

The marked angles form a straight angle, therefore they are supplementary angles which implies their sum is 180°.

Solution(continued):

$$\begin{aligned} 4x - 15 + x - 5 &= 180 && \text{(Supplementary angles)} \\ 5x - 20 &= 180 && \text{(Simplify)} \\ 5x &= 200 && \text{(Simplify)} \\ x &= 40 && \text{(Division Property)} \end{aligned}$$

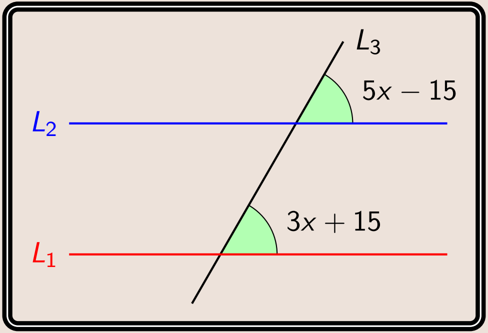
Now to measure each angle, we simply evaluate each one with $x = 40$.

$$\begin{aligned} 4x - 5 &= 4(40) - 5 \\ &= 135, \text{and} \\ x - 5 &= 40 - 5 \\ &= 35 \end{aligned}$$

$\boxed{\text{The marked angle is } 35^\circ \text{ and } 135^\circ.}$

Example:

Assuming $L_1 \parallel L_2$, find the measure of each marked angle.



Solution:

The marked angles are corresponding angles therefore they are congruent.

Solution(continued):

$$\begin{aligned}
5x - 15 &= 3x + 15 && \text{(Corresponding angles)}\\
5x - 3x &= 15 + 15 && \text{(Equation property)}\\
2x &= 30 && \text{(Simplify)}\\
x &= 15 && \text{(Division Property)}
\end{aligned}$$

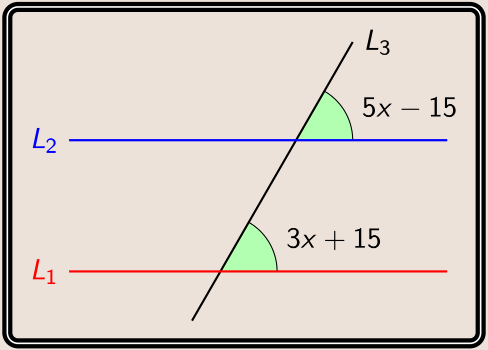
Now to measure each angle, we simply evaluate each one with $x = 15$.

$$\begin{aligned}
5x - 15 &= 3x + 15\\
5(15) - 15 &= 3(15) + 15\\
75 - 15 &= 45 + 15\\
60 &= 60
\end{aligned}$$

The marked angles is 60° each.

Example:

Assuming $L_1 \parallel L_2$, find the measure of each marked angle.



Solution:

The marked angles are corresponding angles therefore they are congruent.

Solution(continued):

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\begin{aligned}
5x - 15 &= 3x + 15 && \text{(Corresponding angles)}\\
5x - 3x &= 15 + 15 && \text{(Equation property)}\\
2x &= 30 && \text{(Simplify)}\\
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\end{aligned}
$$

Now to measure each angle, we simply evaluate each one with $x = 15$.

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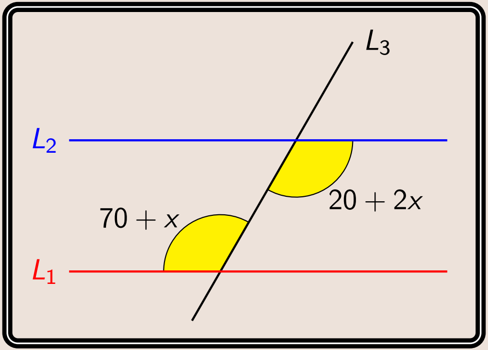
\begin{aligned}
5x - 15 &= 3x + 15\\
5(15) - 15 &= 3(15) + 15\\
75 - 15 &= 45 + 15\\
60 &= 60
\end{aligned}
$$

$\boxed{\text{The marked angles are } 60^\circ \text{ each.}}$

Example:

Assuming $L_1 \parallel L_2$, find the measure of each marked angle.



Solution:

The marked angles are alternate interior angles therefore they are congruent.

Solution(continued):

$$\begin{aligned} 20 + 2x &= 70 + x && \text{(Alternate interior angles)} \\ 2x - x &= 70 - 20 && \text{(Equation property)} \\ x &= 50 && \text{(Simplify)} \end{aligned}$$

Now to measure each angle, we simply evaluate each one with $x = 100$.

$$\begin{aligned} 20 + 2x &= 70 + x \\ 20 + 2(50) &= 70 + 50 \\ 20 + 100 &= 120 \\ 120 &= 120 \end{aligned}$$

The marked angles are 120° each.

Start at ELAC, Go Anywhere